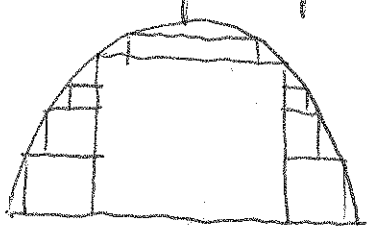
Introduction. Review for Definite integral / Area.

- Goals: Problem solving techniques / Math way of Thinking

- (Definite) Integral:

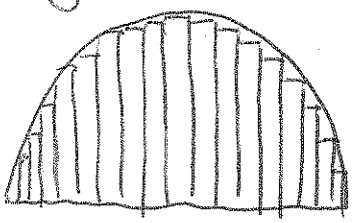
Motivation: How to measure the area of (half) unit disk?

?

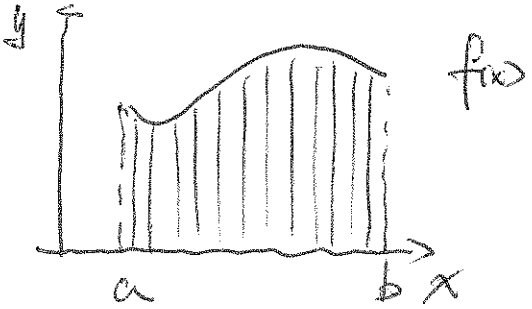


Approximation by rectangles

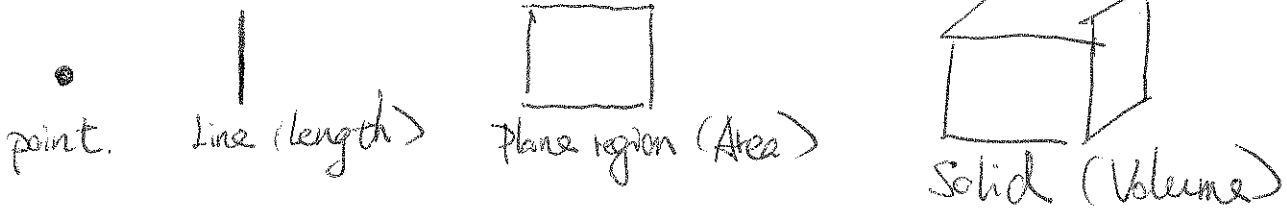
Order / Arrangement.



In general



$$A = \int_a^b f(x)\,dx$$



Differentiation ⟷ Integration

Acceleration Velocity Displacement (Distance)

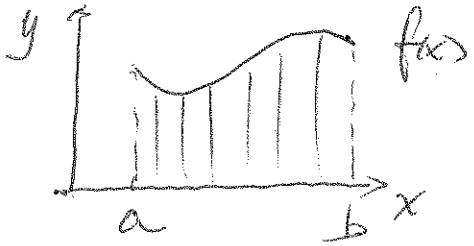
Force. Work.

- See up the integral (Chapt 5, 6)
- Evaluate the integral (Chapt 7)

(Fundamental Theory of Calculus)

$\int_a^b f(x)\,dx = F(b) - F(a)$, where $F(x)$ is an Anti-Derivative of $f(x)$.

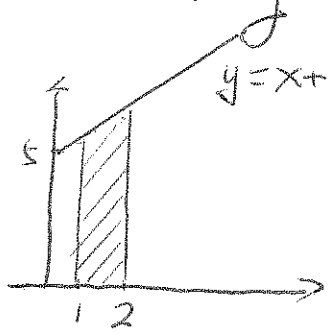
Area Between Curves (Review of §5.1)



$\longleftrightarrow$ Area $= \int_a^b f(x)\,dx$.

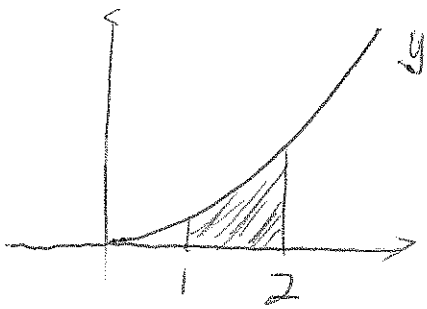
dx suggests "sum up along x-direction".

e.g. $\int_1^2 x+5\,dx$. $\longleftrightarrow$

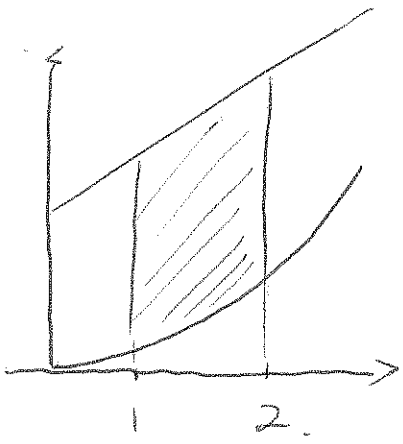


Region I

$\int_1^2 x^2\,dx \longleftrightarrow$



Region II

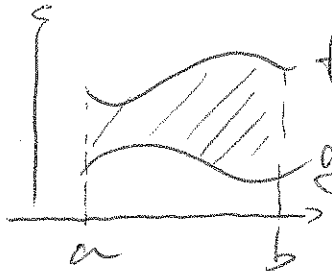


Area = Region I − Region II

$$= \int_1^2 x+5\,dx - \int_1^2 x^2\,dx$$

$$= \int_1^2 x+5-x^2\,dx.$$

- In general.



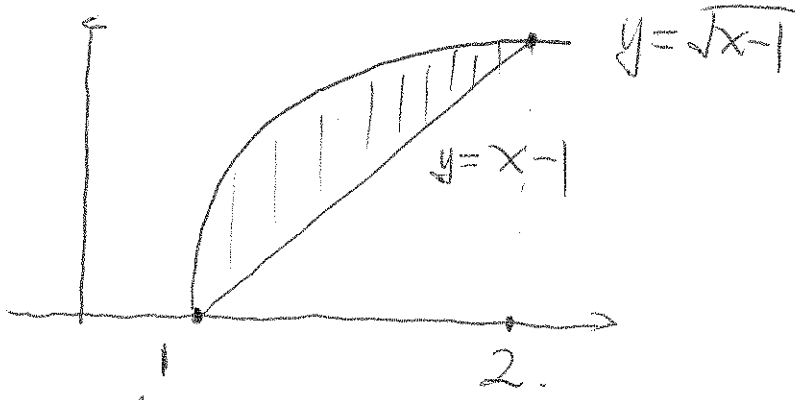
$$\text{Area} = \int_a^b f(x) - g(x)\,dx$$

- e.g. Sketch the region enclosed by the given curves and set up the integral for the area.

$y = \sqrt{x-1}$, $x - y = 1$.

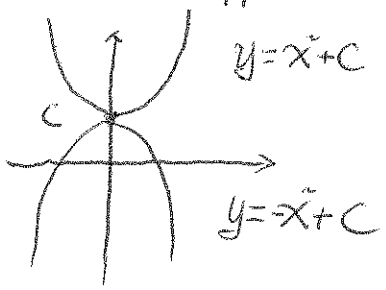
$\Downarrow$

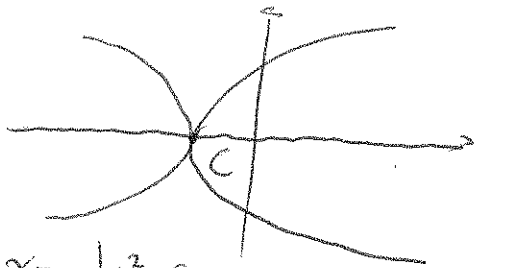
$(y = x-1)$



$$\text{Area} = \int_1^2 \sqrt{x-1} - (x-1)\,dx$$

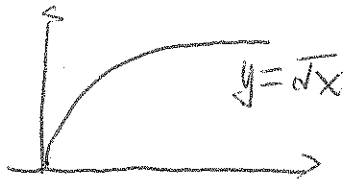
Appendix. graph of frequently used functions

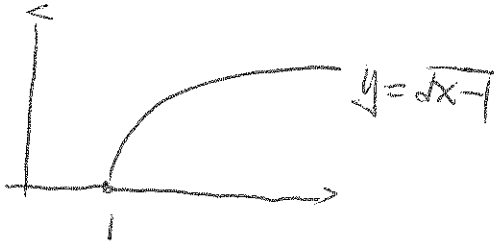
★ 

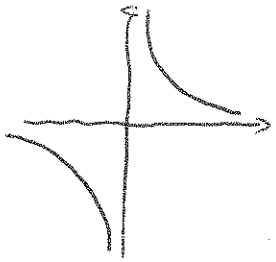
★ 

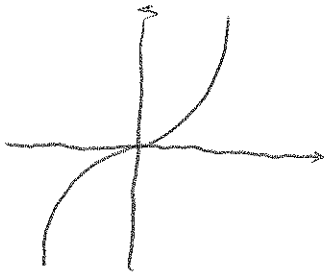
(horizontal and vetical parabola)

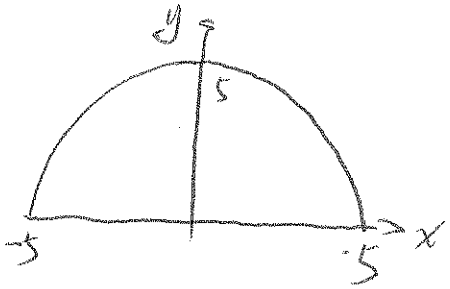
★★ half parabola $y=\sqrt{ax+b}$

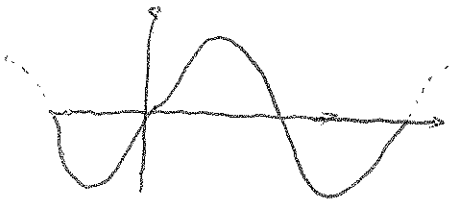
eg. 

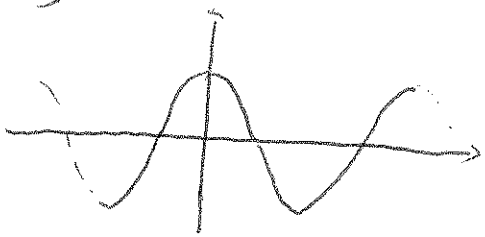


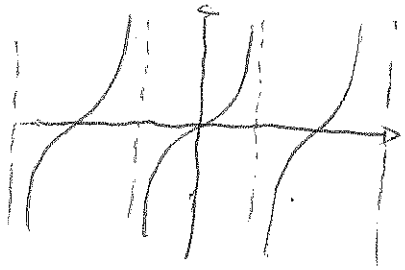
★ $y=\frac{1}{x}$ 

$y=x^3$ 

★ $y=\sqrt{25-x^2}$ (half circle) 

★ $y=\sin x$ 

$y=\cos x$ 

★★ $y=\tan x$. 

★★★ (chapter 6) $y=e^x$, $y=\ln x$.